



Lecture 11: Secure Messaging

CPSC 4440/5440: Real World Cryptography

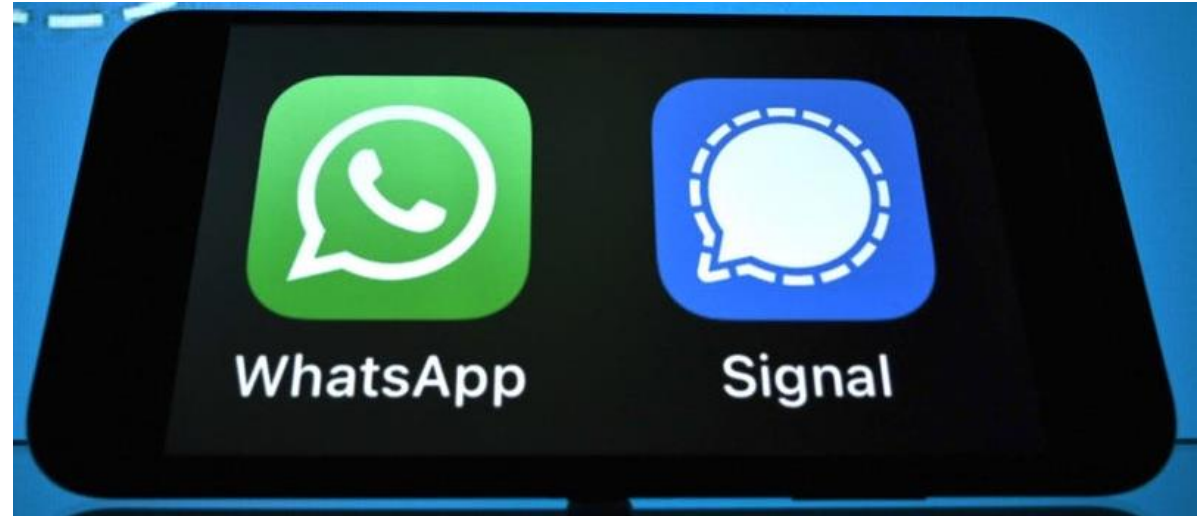
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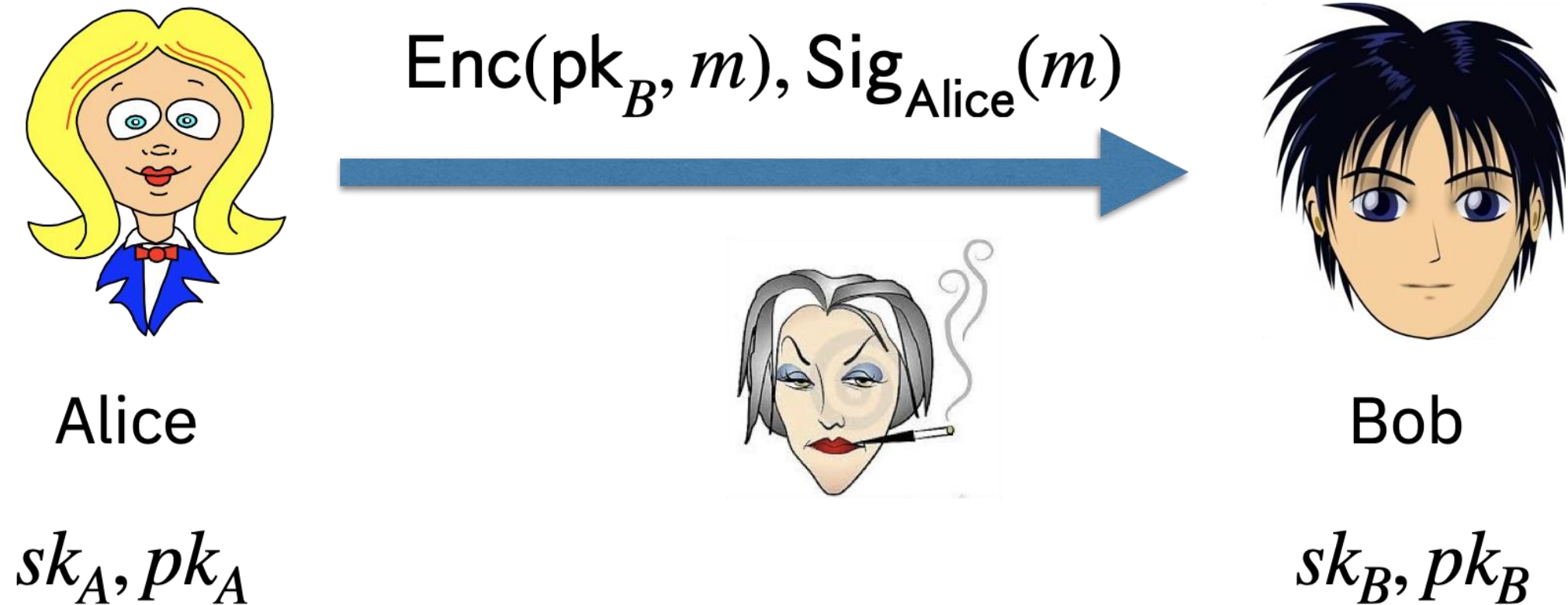


- Have you used any of these?
- Do you know how they work?

The Problem of Secure Messaging

- So far: Metadata hiding against **traffic analysis**
- Today we are going to tackle another threat:
device compromise.
- Let's make generous assumptions
 - They know **how to use public key cryptography**
 - They know each other's **public keys**
 - They don't want to hide metadata, **just content**

Doesn't PKC solve the problem?



Seems *Pretty Good Privacy* for Alice.

Doesn't PKC solve the problem?

Pretty Good Privacy

Original author(s) [Phil Zimmermann](#)

PGP Inc.

Network Associates

[PGP Corp.^{\[1\]}](#)

Developer(s) [Broadcom Inc.](#)

Initial release 1991; 34 years ago

Stable release 11.4.0 Maintenance Pack 2
/ May 23, 2023; 21 months
ago^[2]

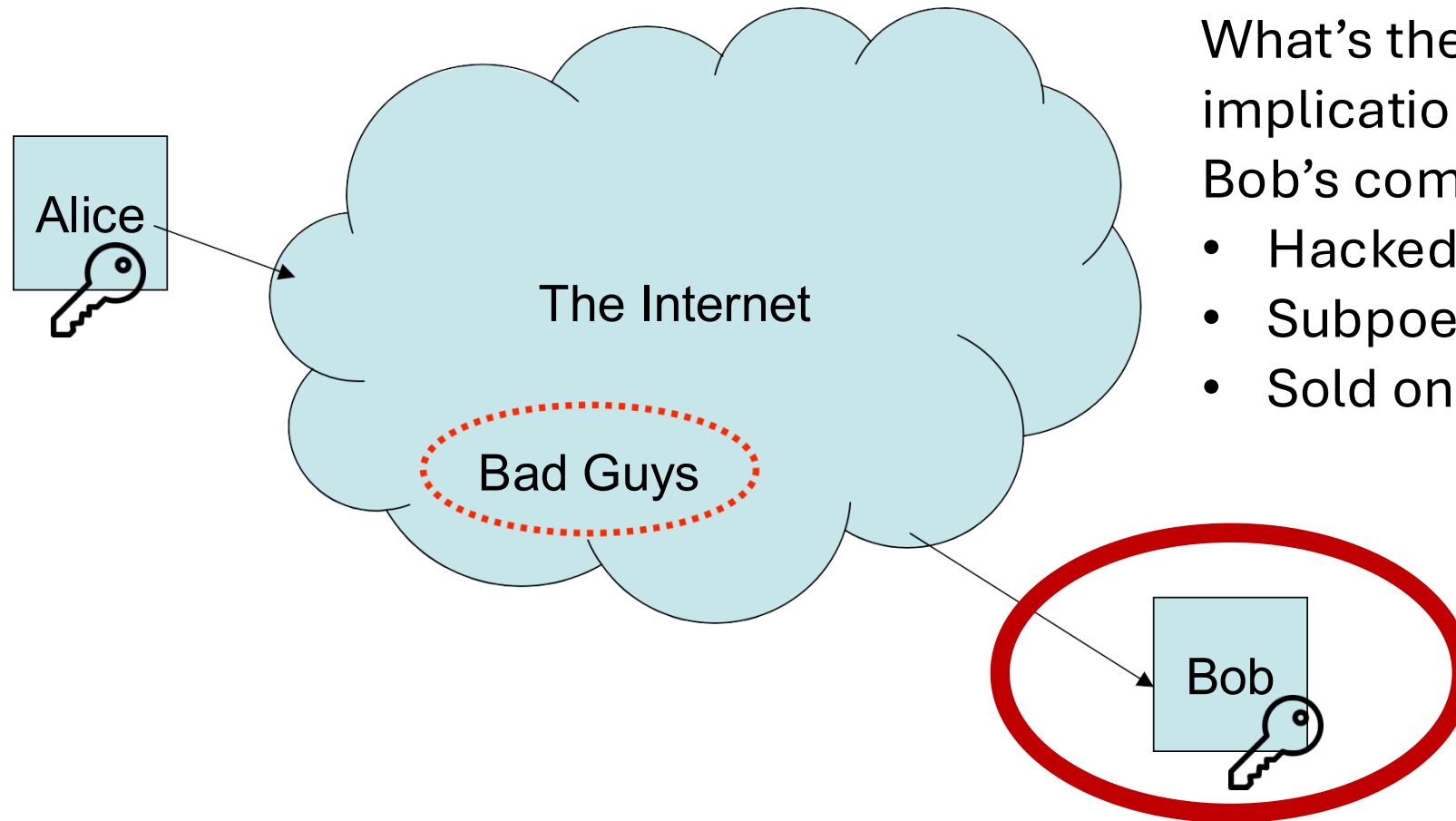


OpenPGP

That's basically how *Pretty Good Privacy* (PGP) works for email encryption.

Why not good enough?

Threat Model



What's the privacy implications for **Alice** if Bob's computer is

- Hacked
- Subpoenaed
- Sold on eBay?

Bad guys who got the keys can...

- Decrypt past messages
- Learn their content
- Learn that Alice sent them
- Obtain a proof about what Alice said.
 - Alice's signatures
- Alice better watches what she says to Bob
 - Her privacy depends on Bob's actions!

What went wrong?

- Bob lets his computer get stolen?
- Have you...
 - left your laptop unattended?
 - delayed installation of security patches?
 - run other's software with sudo?
- What about grandpa?

What Really Went Wrong

- The threat model does not anticipate breaches
- Symptoms in PGP
 - $\Rightarrow$ creates incriminating records of what a party has said
 - $\Rightarrow$ key breach leaks *past* messages
- What are ideal privacy properties for messaging?

Ideal: Casual Conversations

- Alice and Bob talk in a room
- No one else knows what they say
 - Unless Alice or Bob tell them
- No one can prove what was said
 - Not even Alice or Bob
 - Illegal to record conversations without notification
- These conversations are “off-the-record”
- Can we have off-the-record messaging on the Internet?

Off-the-Record Communication, or, Why Not To Use PGP

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Crypto Tools

- What we need is
 - **Perfect forward secrecy (PFS)**
 - **Deniability**

Perfect Forward Secrecy

- We have seen PFS in AKEs: If a server is compromised in the future, *past* communication remain confidential.
- General idea:
 - Use short-lived **encryption keys**
 - **Securely erase** retired keys from memory
 - Use long-term **signature keys** to help distribute & authenticate the short-lived encryption key

Deniability [New]

- There are *many* flavors of deniability.
- E.g., deniability against an eavesdropper:
 - Eve intercepts a communication **transcript T** between Alice and Bob
 - Eve presents **T** to a **judge**
 - Alice and Bob can **deny** their association with T, by saying “Eve created the T all by herself”
- Judge only considers *verifiable evidence*.
- Example of non-deniable protocol: PGP

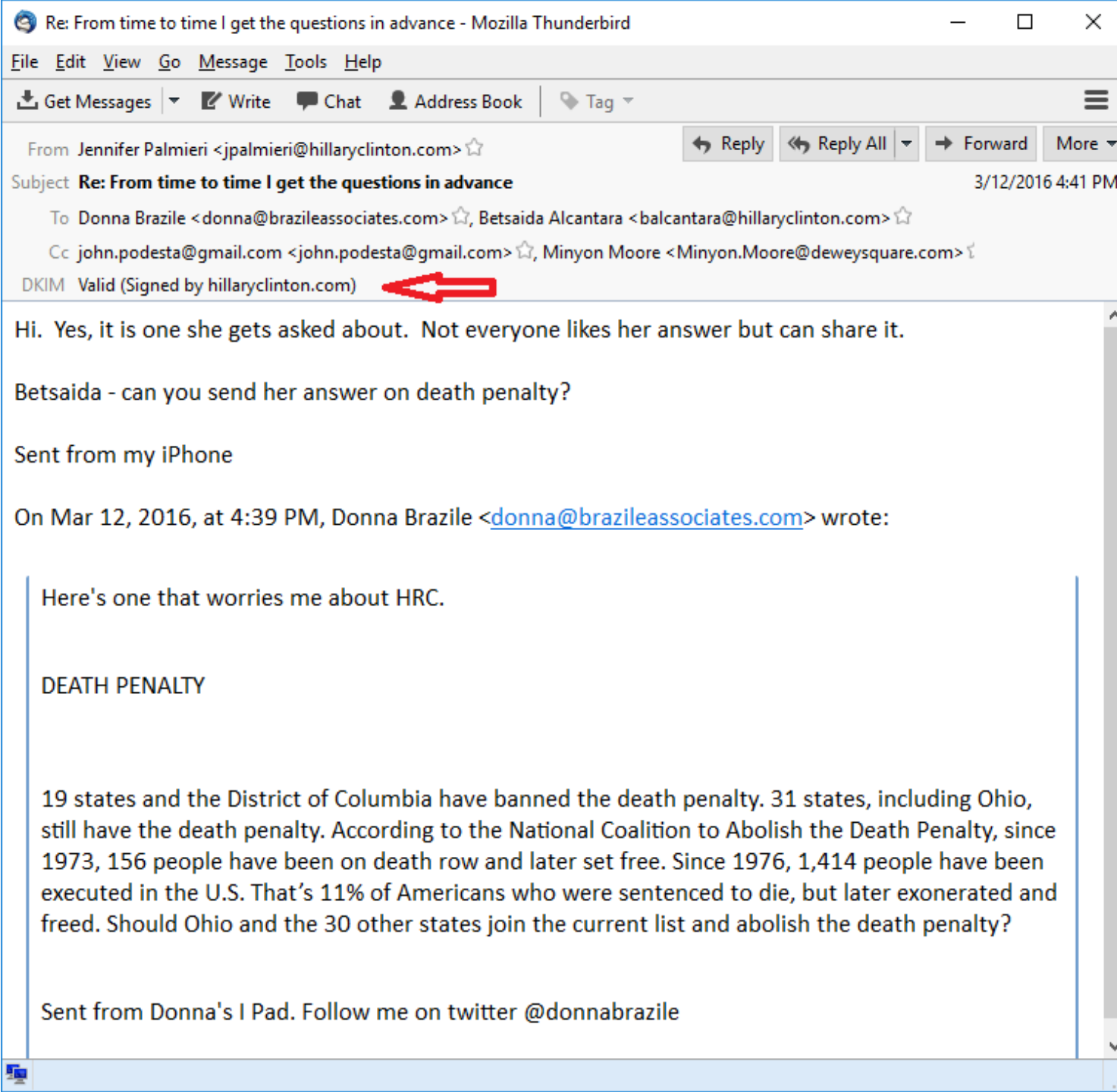
Deniability [New]

- Stronger definition: Bob is malicious.
 - Alice said X to Bob.
 - Bob records the transcript T.
 - Bob presents T to a judge, trying to prove “Alice said X” or even “Alice spoke with me”
 - Deniability for Alice: “Bob could have generated T all by himself.”
- Judge can be **online** or **offline**
 - Online: judge interacts with Bob while Alice and Bob talk
 - Offline: judge appears *after* the conversation

Deniability probably have changed history



- In October 2016, the personal emails of campaign chairman were leaked. These revealed inner campaign strategy and excerpts from Clinton's private speeches to Wall Street firms.



changed history

- Emails are digitally signed by providers to prevent spams
- Called **Domain Keys Identified Mail (DKIM)**
- DKIM signatures undermined deniability by democrats.

Deniability probably have changed history

- 2016: DKIM signatures proved authenticity of some (but not all) emails from Hillary Clinton breach
- 2020: Republican released one email (with DKIM signature) claimed to have been from Hunter Biden's laptop.
- If emails were *off-the-record*, history might look different... Not hard to implement.

Off-the-Record Protocol

- **Security goals:** Confidentiality, Authentication, Perfect Forward secrecy, Deniability
- Assume Alice and Bob know each other's public keys
 - We of course need to deal with that later

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Step 1: Deniable AKE

- OTR original paper proposed *insecure* Diffie-Hellman KE
- Alice and Bob pick random x, y resp.
- $A \rightarrow B: g^x, \text{Sign}_{\text{Alice}}(g^x)$
- $B \rightarrow A: g^y, \text{Sign}_{\text{Bob}}(g^y)$
- Both: use $SS=g^{xy}$ as the shared secret
- Do you spot the security bug?
- **Deniable:** Bob can't prove to a judge that Alice talked with him.

Deniable Authentication

- Need to authenticate msg authorship
 - Do not want to sign messages (non-repudiation)
 - What do use?
-
- Answer: Message Authentication Codes (MACs)

MAC allows Deniability

- Alice and Bob have the same mac key
- Bob cannot prove that Alice generated the MAC
 - He could have done it, too
 - Anyone who can verify can also forge
- Encryption also must be perfect malleable (e.g., AES-CTR mode)

Step 2: Message Transmission

- Both derive Encryption Key (EK) and MAC Key (MK)
 - $EK, MK \leftarrow KDF(SS)$
- $A \leftrightarrow B: (Enc_{EK}(M), MAC(Enc_{EK}(M), MK))$

Step 3: Re-key

- How about PFS?
- To get PFS, Alice and Bob perform a new AKE after every message.

Step 3: Re-key

Can **piggyback DHKE on regular messages** (MACs are omitted)

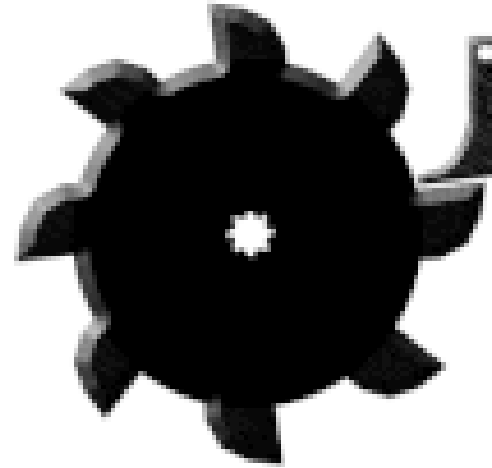
Initial KE 

$A \rightarrow B :$	g^{x_1}	
$B \rightarrow A :$	g^{y_1}	
$A \rightarrow B :$	$g^{x_2}, E(M_1, k_{11})$	Alice: write to me using a new key
$B \rightarrow A :$	$g^{y_2}, E(M_2, k_{21})$	Bob derives $k_{21} = \text{DH}(g^{x_2}, g^{y_1})$
$A \rightarrow B :$	$g^{x_3}, E(M_3, k_{22})$	and so on...

where $k_{ij} = H(g^{x_i y_j})$, the result of a 128-bit hash function

- Alice **securely erases** x_1 after Bob started to use k_{2*}
 - Practical consideration: messages may arrive out-of-order
- If Alice is compromised after that, still can't decrypt k_{1*}

Concept: Key Ratcheting



Property 1: Old keys can't be recovered from new keys

Diffie-Hellman Key Ratcheting

$$A \rightarrow B : g^{x_1}$$

$$B \rightarrow A : g^{y_1}$$

$$A \rightarrow B : g^{x_2}, E(M_1, k_{11})$$

$$B \rightarrow A : g^{y_2}, E(M_2, k_{21})$$

$$A \rightarrow B : g^{x_3}, E(M_3, k_{22})$$



Suppose Alice's compromised at *this* point: leaks k_{2*}



Alice's anti virus killed the trojan at *this* point: secrecy sealed by k_{3*}

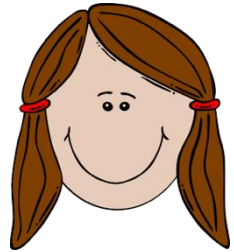
where $k_{ij} = H(g^{x_i y_j})$, the result of a 128-bit hash function

- **Property 2 (Self-healing):** After the attacker is removed, compromise of x_n is *healed* by x_{n+1} . (Same for y .)

Step 4: Publish MK

- For *extra* deniability, Alice and Bob publish old MAC keys.
- How?
 - E.g., Alice sends $(MK_{n-1}, g^{x_n}, E(msg_n))$ to Bob
 - Since the channel is insecure, MK_{n-1} is **plausibly leaked**
- This lets anyone forge messages
 - Provides extra deniability at message level
- Could be added to DKIM, which rotates keys every 3 months

Putting everything together: Recipe for E2EE



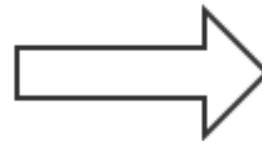
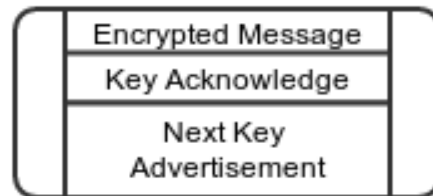
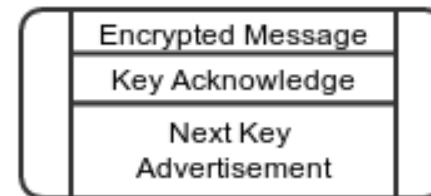
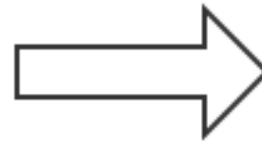
x_0, PK_{Bob}

K_{00}

Deniable
Authenticated Key
Exchange

y_0, PK_{Alice}

K_{00}



Key Ratcheting

Publishing old MAC keys

OTR

- OTR (2004) initiates the study of **secure messaging**
- Features: deniability and forward secrecy
- Modern versions and variants of OTR
 - OTR v4
 - Signal, WhatsApp, Facebook Messenger

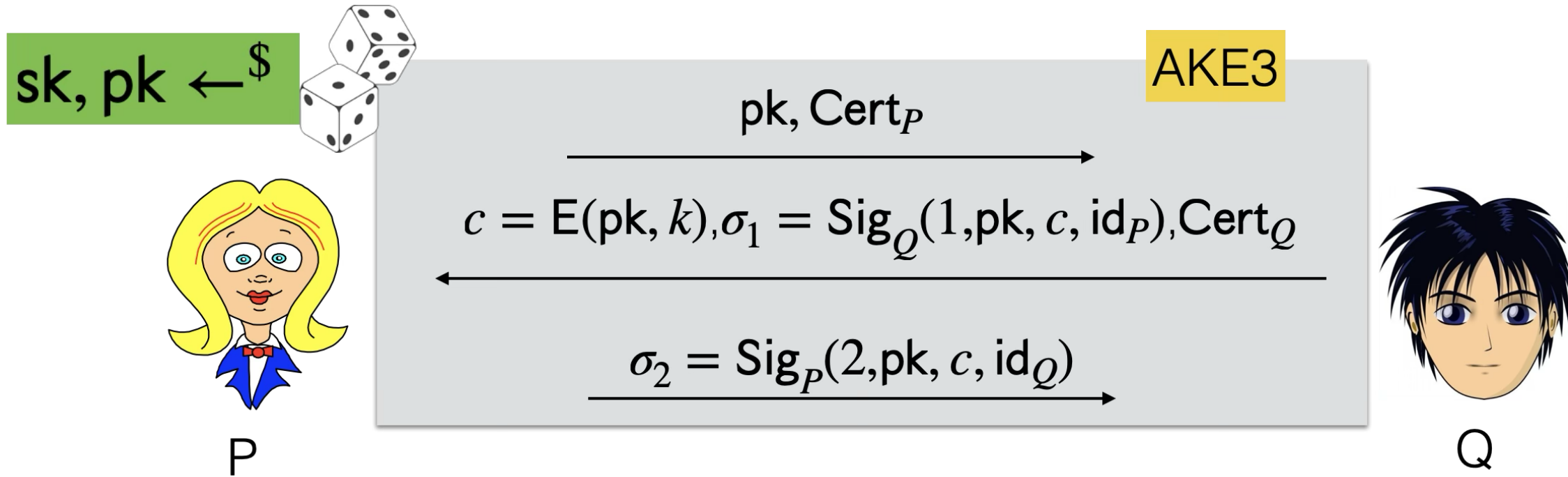


Deniable
Authenticated Key Exchange

Techniques to achieve Deniability

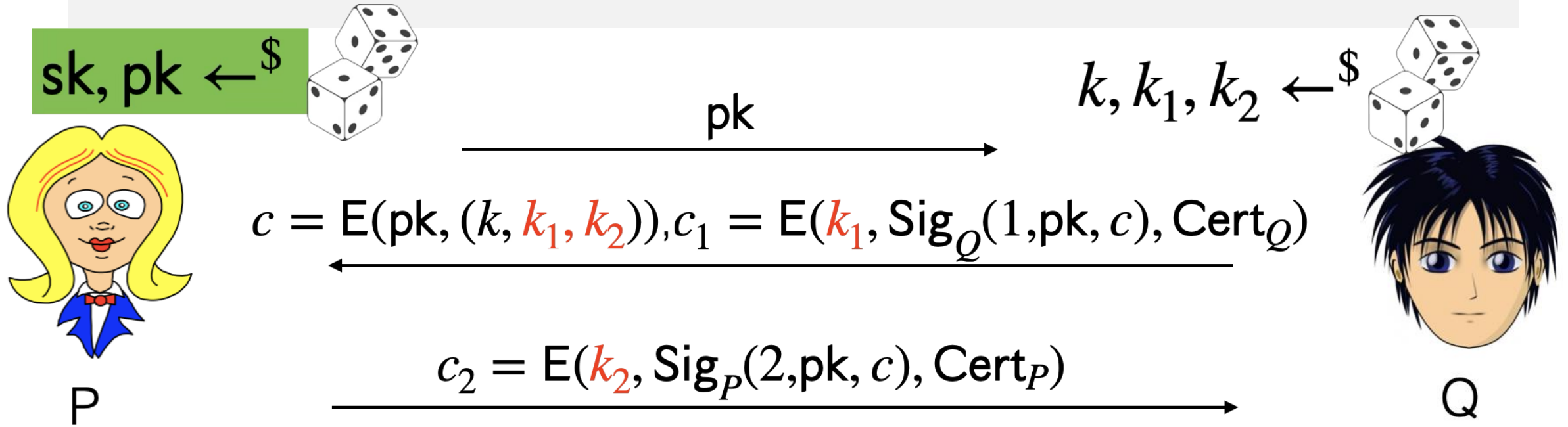
- Deniability for P is ensured by a proof that when P engages in the protocol with Q, whatever evidence an attacker A is able to gather that might implicate P, **A could have generated on its own.**
- Technically: Show a simulator S who can generate transcripts indistinguishable from real conversation
 - without talking to P,
 - or only involving P.
- Conceptually similar to how we show zero-knowledge of Schnorr ID protocol.

Warm-up: AKE3 from Lecture 3



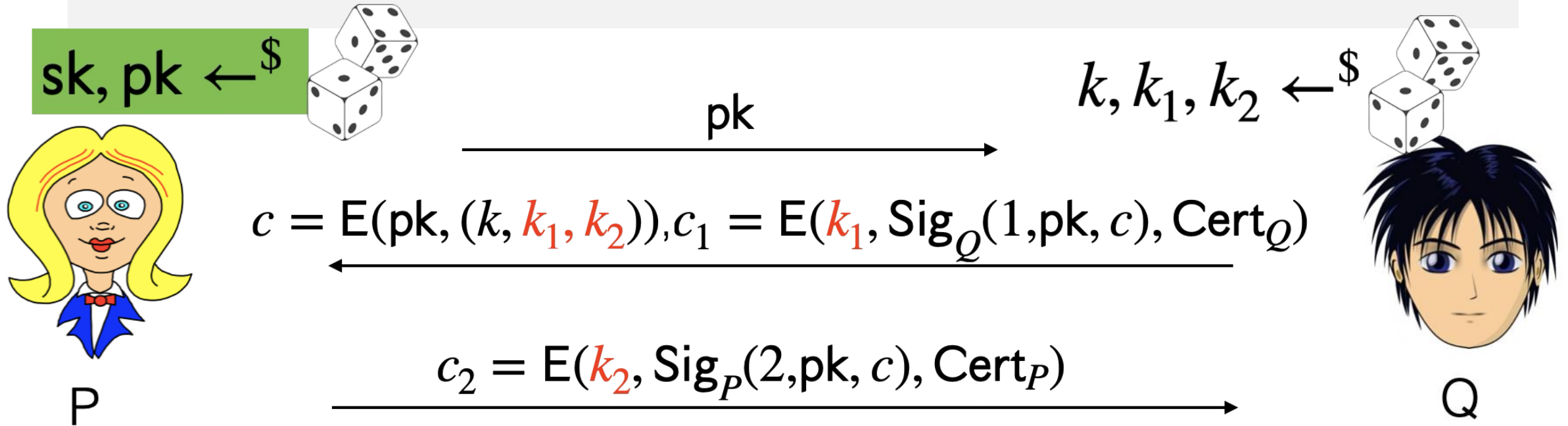
Warmup: Is AKE3 deniable by P (against Eavesdropper or malicious Q)?
What about Q?

AKE4



- Is AKE4 deniable by P against an eavesdropper?

AKE4



- Is AKE4 deniable by P against a **malicious Q** ?
- Proof: P can say: “I was talking with someone else.”
- Q ’s attack: choose $k = H(\text{Sig}_Q(pk))$.

AKE5 (Nobody signs anything)



P

Public Key $A = g^\alpha$

$$u \xleftarrow{\$} Z_q$$

$$U = g^u, Cert_P$$



Q

Public Key $B = g^\beta$

- $v \xleftarrow{\$} Z_q$
- $k_1, k_2, k_3 = H((AU)^{\beta+v}, \dots, id_P, id_Q)$

$$V = g^v, k_1, Cert_Q$$



$$k_1, k_2, k_3 = H((BV)^{\alpha+u}, \dots, id_P, id_Q)$$

k_2



Check against k_1 received

Check against own k_2

Use k_3 as session key

Use k_3 as session key

Deniability argument (intuition)

Public Key g^α (Q doesn't have this secret key)

Public Key g^β

$$u \leftarrow^{\$} Z_q$$

$$U = g^u, Cert_P$$



Q

Q can generate an identically distributed transcript by himself without talking to P.

$$g^v, k_1, Cert_Q$$

$$v \leftarrow^{\$} Z_q$$

$$k_1, k_2, k_3 = H((AU)^{\beta+v}, \dots)$$

$$k_2$$

Highlights of Signal Protocol

A bit history about



- Introduced in 2010 (then TextSecure)
- Acquired by Twitter in 2011
- In 2013, key people left Twitter and founded Open Whisper Systems. Fully open-sourced TextSecure.
- Introduced ***Double Ratcheting***
- Renamed to Signal in 2014
- The **Signal protocol** is used by WhatsApp in 2016 for its billions of users.
- As of January 2022, the Signal app had approximately 40 million monthly active users [Wikipedia]

Signal's X3DH

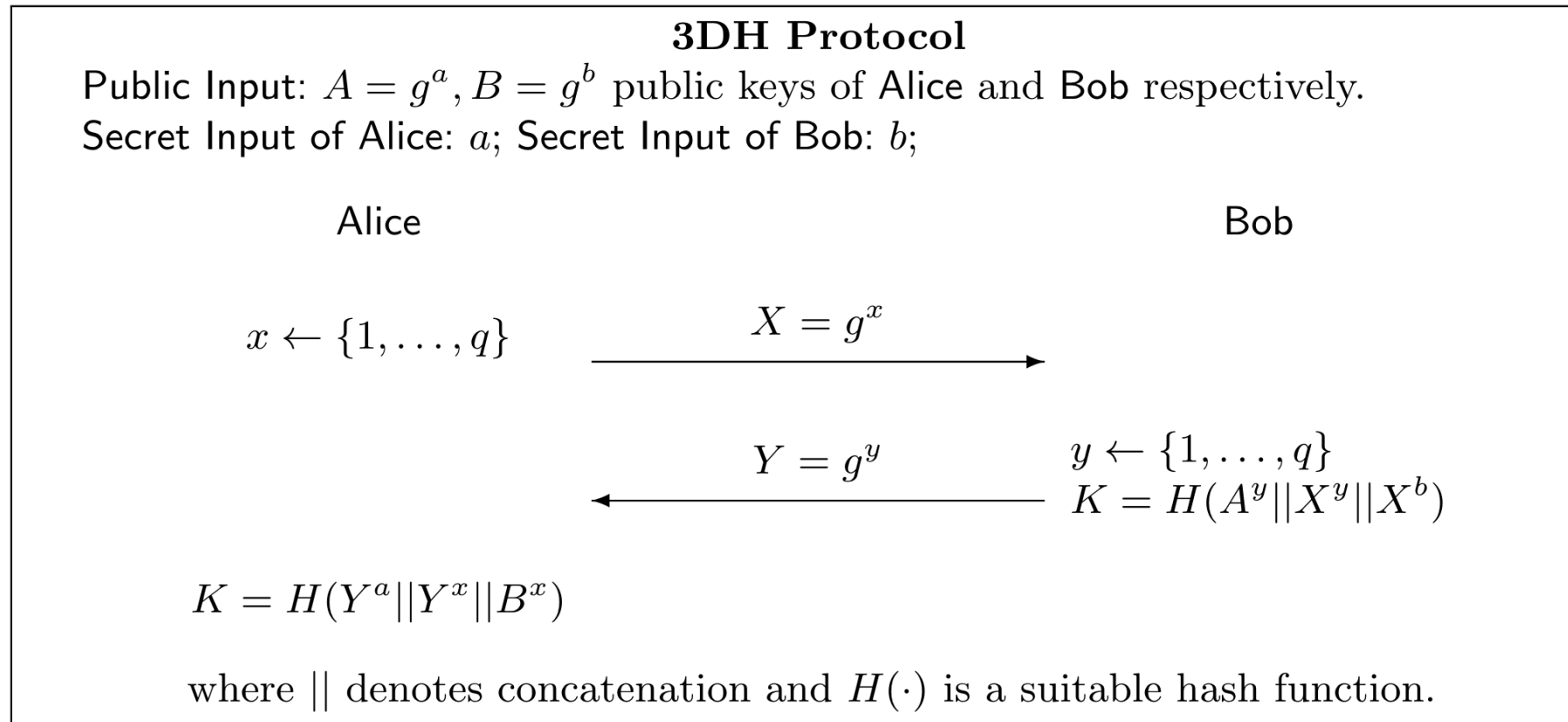


Fig. 2. Triple Diffie-Hellman key exchange protocol

Signal's X3DH

3DH Protocol

Public Input: $A = g^a, B = g^b$ public keys of Alice and Bob respectively.
Secret Input of Alice: a ; Secret Input of Bob: b ;

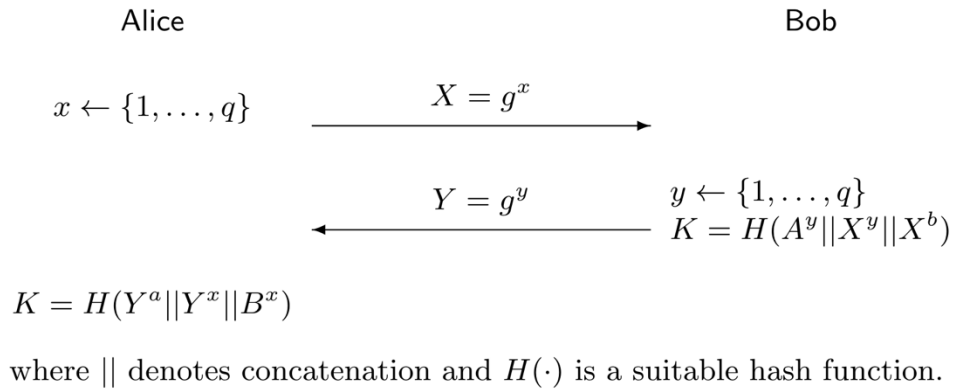
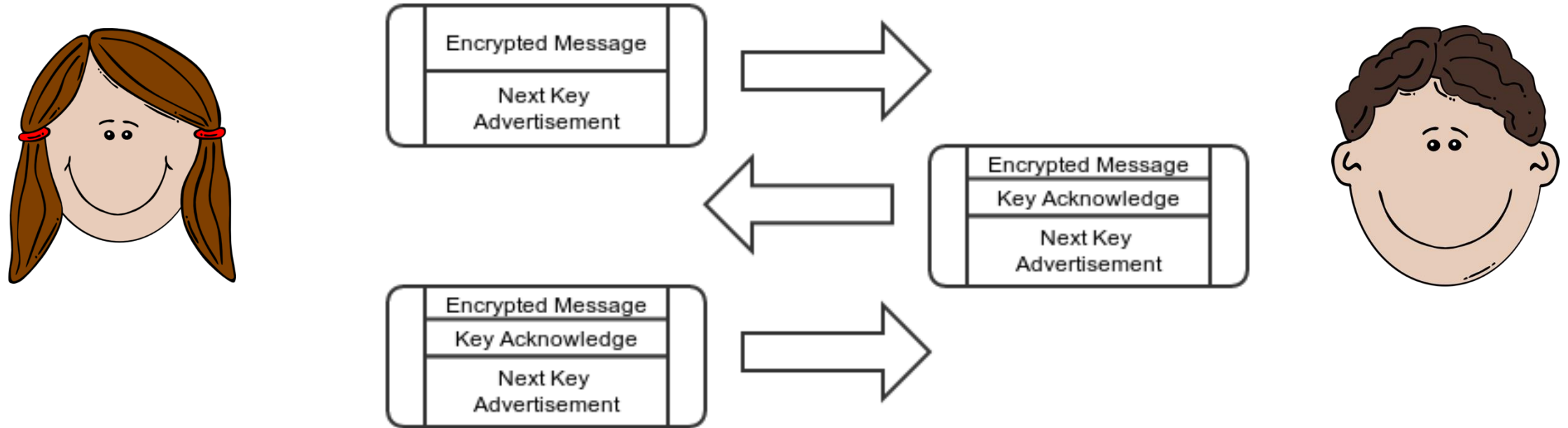


Fig. 2. Triple Diffie-Hellman key exchange protocol

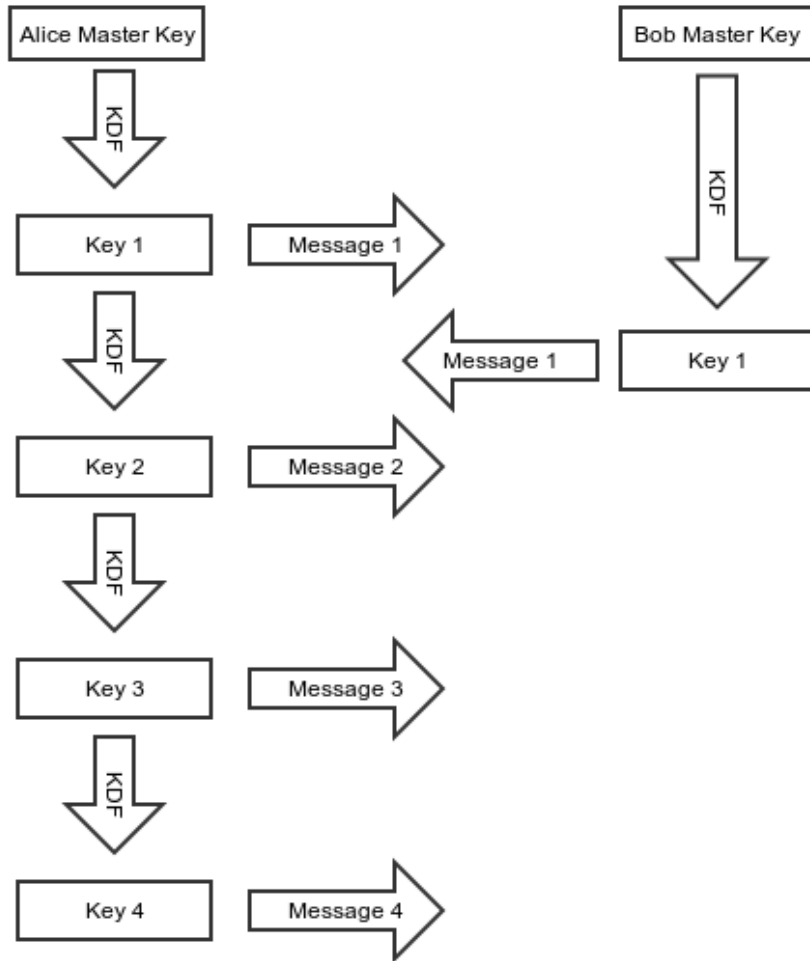
- Alice and Bob have long term keys g^a, g^b and short-term keys g^x, g^y .
- $K = H(g^{ya} || g^{xy} || g^{xb})$
- **Authenticity:** an MiTM attacker must know **a and x**, or **b and y**, or **x and y**
- **Full Participation Repudiation:** Intuitively, anyone can forge a session between Alice and Bob (generate random x, y)
- Proof that any transcript can be *simulated* is non-trivial [ANCS20].

Signal's observations about DH Ratcheting



- Seems an **overkill** to redo AKE between every message (key compromise unlikely)
- Make sense to reduce cadence (e.g., every change of sender, or first message after long silence)
- How to refresh keys in between DH Ratcheting?

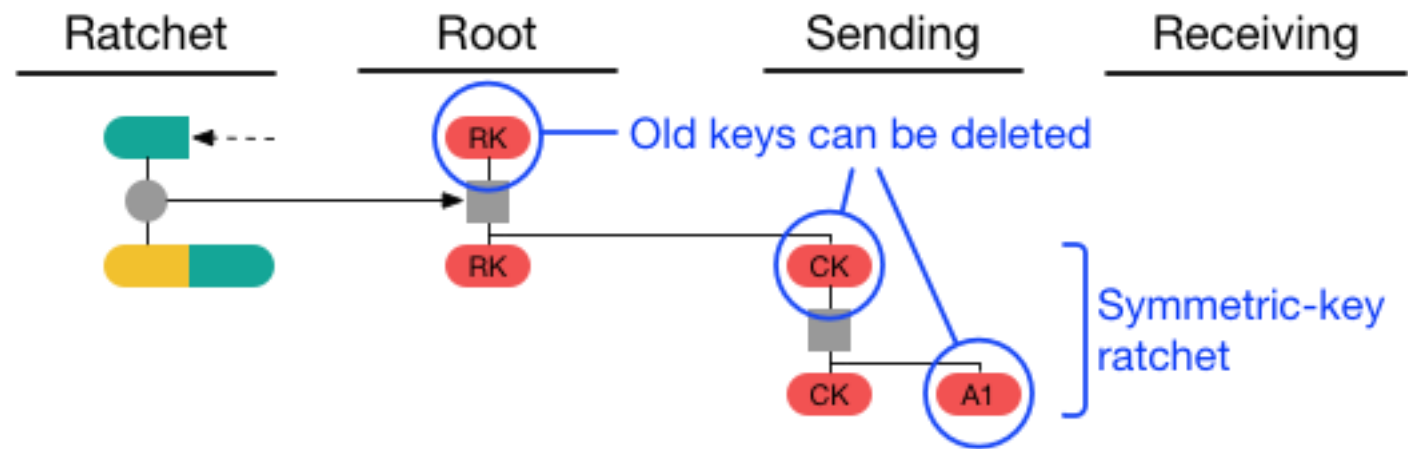
Alternative: hash chain



- KDF ratchet:
$$\text{Key}_k = \text{hash}^k(\text{Key}_0)$$
- I.e., repeated hash
- Upsides:
 - Forward secrecy (can't invert hash func.)
 - Computationally light weight
 - non-interactive
 - Handling of out-of-order delivery is easier: msgs are attached with seq# (can "skip")
- Downsides: not self-healing

Signal's Double Ratcheting Algorithm

- Combines **hash chain** and **DH ratchets**
- For Alice: DH ratchet is advanced every time Bob replies; before that, each msg advances the hash chain ratchet.
- Two ratchets: send/receive
- Handles out of order msg



A problem hidden under the rug: Trust establishment

- How do Alice know that she is communicating with Bob?
- Signal uses



Safety Numbers

- We will study *Key Transparency Logs* in the next chapter.

Summary: Secure Messaging

- Important concepts pioneered by OTR: **forward secrecy, post-compromise security (self-healing), deniability**
- Secure messaging protocol typically involves
 - Deniable Authenticated Key Exchange
 - Key ratcheting algorithms
 - Releasing the MAC key if necessary
- Signal is inspired by ORT, with various features
 - Asynchronous msgs
 - out-of-order msgs
 - Verification via Safety Numbers
 - Social discovery
- Under the rug: trust establishment